SAVIAC

(2)

Proceedings of the 62nd Shock and Vibration Symposium

Volume II

Hosted by the
Defense Nuclear Agency

29-31 October 1991
Springfield, Virginia

92 4 24 150

TIME HISTORY ANALYSIS OF SYSTEMS AS AN ALTERNATIVE TO A DDAM-TYPE ANALYSIS

G.J. O'Hara and P.F. Cunniff
Department of Mechanical Engineering
University of Maryland
College Park, Md. 20742

This paper reports on the degree of success that may be achieved by using simple equipment-vehicle models that produce time history responses which are equivalent to the responses that would be achieved using spectral design values employed by the Dynamic Design Analysis Method. The equipment models studied herein are limited to one and two-degree of freedom systems; the vehicle to which the equipment is attached consists solely of a rigid mass; and the shock excitation is produced by an ideal impulse that is applied to the vehicle mass so as to produce an initial velocity. Although the case of the single-degree of freedom equipment presents no difficulties in performing a transient analysis that reproduces the DDAM-like response, it is shown that there are no unique values for the vehicle mass and for the magnitude of the impulse, but that they are interrelated. In the case of the two-degree of freedom equipment, the transient analysis duplicates the response that would be experienced using the DDAM-like input values, provided the shock design value in the first mode is less than the shock design value in the second mode. Otherwise, solutions are not possible. Brief comments are also provided for the special case of an equipment composed of a very light mass attached to a large mass, and for the case of an equipment with repeated natural frequencies.

INTRODUCTION

The Dynamic Design Analysis Method (DDAM) [1] has been used for the past 30 years as part of the Navy's efforts to shock-harden heavy shipboard equipment. This method, which has been validated several times [2], employs normal mode theory [3,4] and design shock values

[5]. Current DDAM practice prescribes a modal analysis approach that utilizes these shock design values in three orthogonal directions and takes into account the type of vehicle, equipment location, i.e., hull-mounted, deck-mounted, and shell-plate mounted.

Since the introduction of DDAM, different transient analysis methods have been proposed as alternative approaches to spectral analysis. One such approach [6] uses a simple base mass to represent the vehicle to which the equipment is attached, and an impulsive force applied to this base mass so as to produce shock excitation. While the transient analysis treatment of shock-excited structures appears attractive, especially since the advent of powerful personal computers, many methods suffer shortcomings in that they are expensive because they are sensitive to both target geometry and the vehicle-equipment model. Although the shock experienced by an attacked equipment-vehicle system is unique, each time the target geometry is changed, the effects are not unique. For example, the maximum stress in a cantilever beam occurs at the root of the beam even though the shock intensity and attack geometry may vary.

Recently, papers have appeared that provide an overview on the evolution of spectral techniques in naval shock design [7]; guidance on accounting for structural interactive effects in choosing design shock values from shock spectra [8]; and the demonstration of a procedure [9] for establishing shock design curves for spectral analysis from accumulated field data.

This paper examines the degree of success that may be achieved by simple equipment-vehicle models in producing the equivalent of DDAM interim inputs [5] by means of impulse response transient analysis. The vehicle is represented by a solid, rigid mass, while the equipment is limited to one or two-degrees of freedom.

BACKGROUND

Suppose an equipment is attached to two different vehicles labeled Vehicle A and Vehicle B, as shown in Figure 1. Assume that each vehicle is subject to a transient excitation and the shock spectra generated from the response at the point of attachment of the equipment to the base of each vehicle is as shown in Figure 2. There are several important observations to be made from Fig. 2. The peaks in the spectra refer to the natural frequencies of the system which is composed of the equipment and the vehicle. These frequencies differ for each system since vehicle A is different from vehicle B. Consequently, the time history transient response from which the shock spectra were developed will be different for each system, as well as the internal equipment response. The second observation is that the shock design values are those values that correspond to the fixed base frequencies f_1 and f_2 of the equipment. Finally, although the modal effective masses and the fixed base frequencies f_1 and f_2 are the same for each system in Fig. 1, the magnitude of the shock design values for the equipment are slightly different for each system because of the different interaction structural characteristics of vehicles A and B. The shock design values are also labeled in Fig. 2, where these least upper bound values are established based upon all available data for the equipment under consideration as explained in reference [9].

Next, consider Fig. 3(a), which shows an equipment attached to a vehicle that is subject to a shock excitation. Knowing the equipment modal effective masses and the fixed base frequencies represented by modal oscillators in Fig. 3(b), we can calculate the set of modal

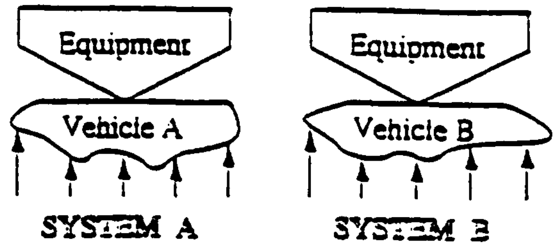

Fig. 1 - An equipment attatched to two different vehicles.

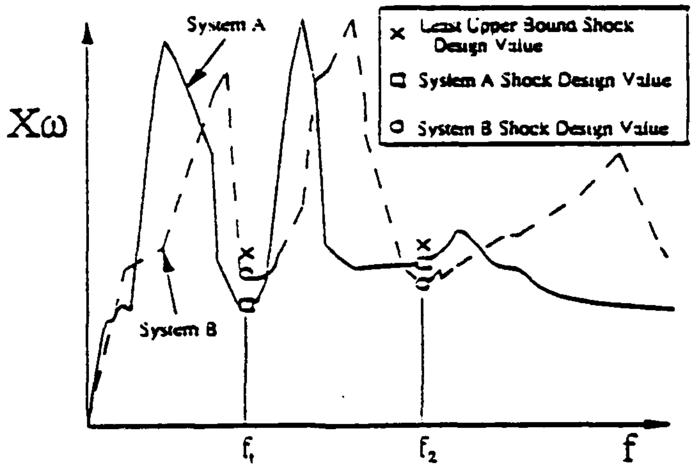

Fig. 2 - Shock spectra for systems A and B and their respective design values relative to the least upper bound shock design values.

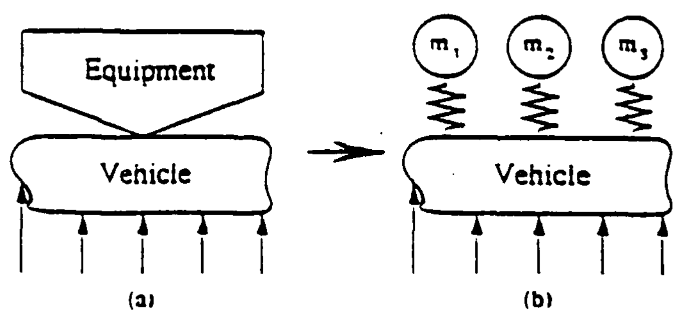

Fig. 3 - Equipment modeled by its modal oscillators.

shock design values N from design curves, such as those shown in Fig. 4. This is essentially the DDAM approach for estimating the maximum deflections and stresses experienced by the equipment using modal analysis. The shock design curves represent a compilation of shock data as described in reference [9]. Each design curve has a limiting maximum pseudo-velocity V_{max} that establishes the slope of the line in Fig. 5 for a given modal weight until the corner frequency f_c is reached. For fixed base frequencies $f > f_c$, the maximum acceleration N_{max} controls the design analysis, where

$$f_c = N_{max} g / (2\pi V_{max}) \tag{1}$$

Suppose the vehicle is replaced by a simple mass m_o and a shock environment that is produced by an impulse of magnitude I_o applied to m_o as shown in Fig. 6. Furthermore, assume that the shock design inputs represented by Fig. 4 still apply to the new system in Fig. 6. The question arises as to whether a transient analysis can be performed on the new system so as to provide the equivalent maximum deflections and stresses in the equipment as those that are found using the spectral analysis approach. In other words, is it possible to find the vehicle mass m_o and the impulse I_o so as to produce the same peak accelerations of the equipment masses as those prescribed by the spectral design values? The following provides a few examples that may shed some light on the viability of using the transient approach.

EXAMPLE 1

The first example represents the equipment by a simple mass m_1 and spring modulus k, as shown in Fig. 7. Assuming that the system is excited by the impulse I_o applied to m_o, it has been shown that the shock design value [10] is given by:

$$\frac{\alpha X}{V_o} = \frac{1}{\sqrt{(1 + \mu)}} \tag{2}$$

where
$$\begin{aligned} X &= Y - Z \\ \alpha &= \sqrt{K/m_1} = \text{equipment fixed base frequency (rad/s)} \\ V_o &= I_o/m_o \\ \mu &= m_1/m_o \\ W_1 &= m_1 g \end{aligned}$$

If we were to perform a DDAM-like analysis on this model, the shock design value would be obtained from Fig. 4 using the design curve that corresponds to W_1, where $\alpha X = Ng/\alpha$. Thus, the effective modal force equal to $W_1 N$ would be applied to m_1 in order to calculate the maximum relative displacement.

Consider performing a transient analysis that will produce the same maximum relative displacement. We observe from Fig. 4 that αX is fixed from the appropriate shock design curve. Thus, we have the dilemma that there is no unique value of m_o to perform the transient analysis; i.e., as we vary V_o in Eq. (2), the vehicle mass m_o also changes. Consequently, although a transient analysis can reproduce the result of the spectral analysis, there is no unique value of m_o that must be used. Note also, that as the value of m_o is changed, the time history of the transient response also changes because of the change in the system frequency.

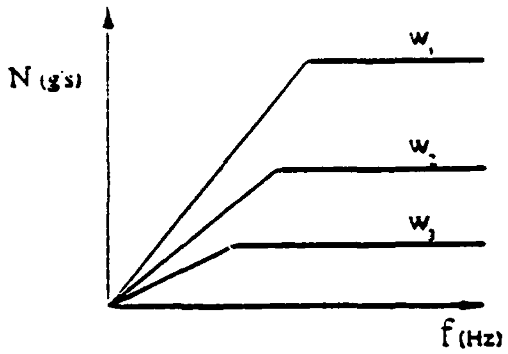

Fig. 4 - Typical shock design curves for modal weights W_1, W_2, and W_3, as a function of the fixed base frequencies.

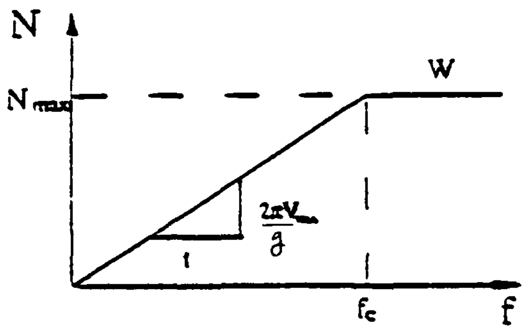

Fig. 5 - Shock design curve for modal weight W showing cut-off frequency f_c, V_{max}, and N_{max}.

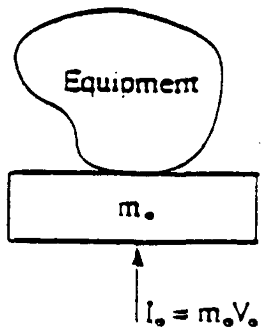

Fig. 6 - Equipment attached to a rigid mass vehicle M_o.

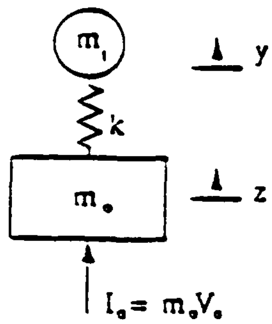

Fig. 7 - Example 1.

EXAMPLE 2

Next, consider a two-degree of freedom equipment represented by its two modal oscillators attached to the vehicle mass m_0, as shown in Fig. 8, where m_1 and m_2 are the modal effective masses for the equipment. The system is excited by the impulse I_0 applied to the vehicle mass m_0. The design shock values can be written as [10]:

$$\frac{\beta X_1}{V_o} = \frac{1 + q\sqrt{1+\theta}}{\sqrt{1+\theta}\sqrt{1+\theta - \frac{\theta}{1+\sigma} + q^2\left(1+\frac{\theta}{1+\sigma}\right) + 2q\sqrt{1+\theta}}} \tag{3}$$

$$\frac{\gamma X_2}{V_o} = \frac{\sqrt{1+\theta} - q}{\sqrt{1+\theta}\sqrt{1+\theta - \frac{\theta}{1+\sigma} + q^2\left(1+\frac{\theta}{1+\sigma}\right) - 2q\sqrt{1+\theta}}} \tag{4}$$

where
$\beta = \sqrt{k_1/m_1}$ = equipment fixed base frequency (rad/s)
$\gamma = \sqrt{k_2/m_2}$ = equipment fixed base frequency (rad/s)
$\theta = (m_1 + m_2)/m_o$
$q = \beta/\gamma$
$\sigma = m_2/m_1$

The ratio of the shock design values reduces to

$$r = \frac{\gamma X_2}{\beta X_1} = \frac{(\phi - q)\sqrt{(\phi+q)^2 - (\phi^2-1)(1-q^2)/(1+\sigma)}}{(1+q\phi)\sqrt{(\phi-q)^2 - (\phi^2-1)(1-q^2)/(1+\sigma)}} \tag{5}$$

$$\text{where } \phi = \sqrt{1+\theta}$$

Since the shock design values βX_1 and γX_2 can be obtained from DDAM-like inputs in ref. [5], the value of r is fixed. Also, q and σ in Eq. (5) are also fixed by the equipment modal characteristics. Thus, the value of m_0 is uniquely established provided that the value of ϕ can be found. In that case, note that the magnitude of V_0 can then be found from either Eq. (3) or Eq. (4).

Table 1 summarizes the characteristics for four cases studied, where the modal weights are 60 kips and 26 kips, respectively. The fixed base frequencies $f_1 = \beta/2\pi$ and $f_2 = \gamma/2\pi$ of the modal oscillators representing the equipment were arbitrarily varied with respect to the shock design cut-off frequencies f_{c1} = 64.9 Hz and f_{c2} = 87.5 Hz as follows:

System 1: f_1 = 58 Hz < f_{c1}, f_2 = 80 Hz < f_{c2}

System 2: f_1 = 58 Hz < f_{c1}, f_2 = 90 Hz > f_{c2}

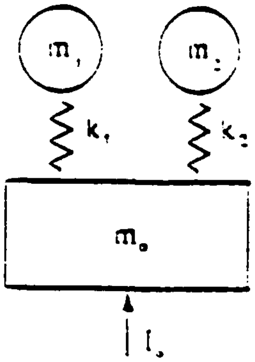

Fig. 8 - Example 2.

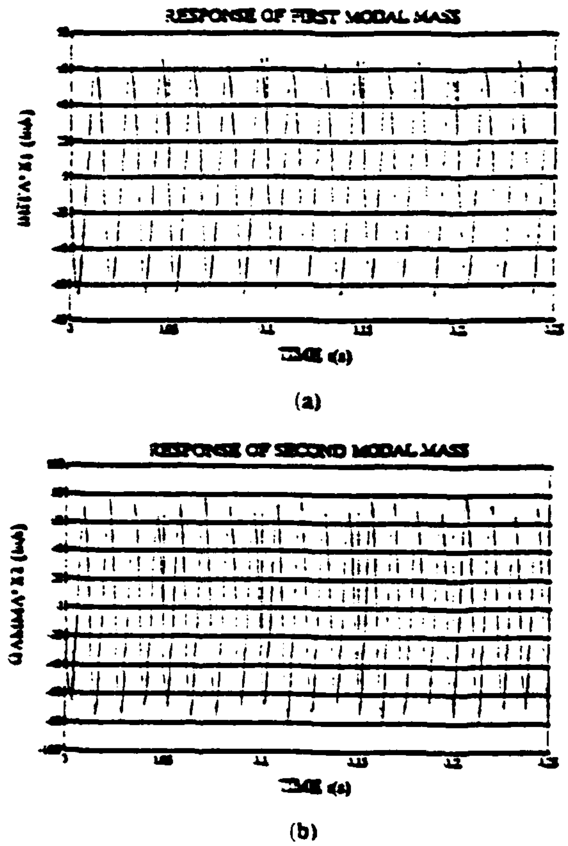

Fig. 9 - Transient response of modal oscillators, Example 2.

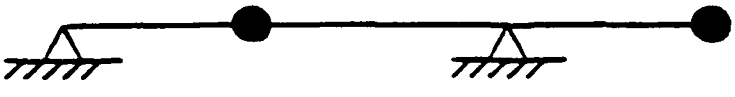

Fig. 10 - Example of a structure where the second modal effective weight exceeds the first modal effective weight.

System 3: $f_1 = 79$ Hz $> f_{c1}$ System 4: $f_1 = 70$ Hz $> f_{c1}$
$f_2 = 90$ Hz $> f_{c2}$ $f_2 = 80$ Hz $< f_{c2}$

The values W_o are also listed in Table 1 as a consequence of solving eq. (5) for each system.

TABLE 1 - System Characteristics, Example 2

	System			
	1	2	3	4
f_1 (Hz)	58	58	70	70
f_2 (Hz)	80	90	90	80
W_1 (kips)	60	60	60	60
W_2 (kips)	26	26	26	26
W_o (kips)	964.012	866.033	859.751	1432.060
V_o (in/s)	70.592	70.993	64.734	63.470

The value of V_o for each system, derived from Eqs. (3) or (4), is also listed in Table 1. Table 2 lists the DDAM-like values for βX_1 and γX_2.

TABLE 2 - Summary of Results, Example 2

System	βX_1 (in/s)		γX_2 (in/s)	
	DDAM	Transient	DDAM	Transient
1	67.5	67.5	80.3	80.1
2	67.5	67.5	78.1	77.7
3	61.6	61.6	78.1	77.9
4	61.6	60.8	80.3	79.8
*4	61.6	61.2	80.3	80.3

These peak values are compared with the corresponding values obtained from the transient analysis represented by the following equations [10]:

$$X_1 = -\frac{V_o}{\omega_2^2-\omega_1^2}\left[\frac{\gamma^2-\omega_1^2}{\omega_1}\sin\omega_1 t + \frac{\omega_2^2-\gamma^2}{\omega_2}\sin\omega_2 t\right] \qquad (6)$$

$$X_2 = -\frac{V_o}{\omega_2^2-\omega_1^2}\left[\frac{\beta^2-\omega_1^2}{\omega_1}\sin\omega_1 t + \frac{\omega_2^2-\beta^2}{\omega_2}\sin\omega_2 t\right] \tag{7}$$

where ω_1 and ω_2 are the system natural frequencies ($\omega_0 = 0$ for the free-free system). Figure 9 shows the transient responses of βX_1 and γX_2 for case 3, which was typical of the four systems. Note that the magnitude V_0 can be adjusted in each case to improve these results. For example, multiplying the original V_0 by (80.3/79.8) would improve the results for System 4, as shown on the line marked * in Table 2.

EXAMPLE 3

Suppose the modal effective weights used in Example 2 are reversed so that $W_1 = 26$ kips and $W_2 = 60$ kips. The overhanging beam in Fig. 10 is an example where the second modal effective mass will be greater than the first modal effective mass. Suppose the fixed base frequencies for the two oscillators are $f_1 = 58$ Hz and $f_2 = 61$ Hz, respectively. The DDAM-like inputs for this case are $\beta X_1 = 80.3$ in/s and $\gamma X_2 = 67.5$ in/s, so that the ratio r is less than one.

Substituting these values into Eq. (5), along with the values for q and σ, we discover that a solution for ϕ cannot be found. Consequently, it appears that whenever $r \leq 1$, it is not possible to find the values of m_0 and V_0 for this model in order to carry out a transient solution that replicates the DDAM-like inputs.

CONCLUDING REMARKS

This somewhat limited examination of transient analysis as a replacement of spectral analysis which is currently employed in DDAM-like analyses shows some promise and some pitfalls. While the single-degree of freedom equipment presents no difficulties in performing a transient analysis using a simple mass to represent the vehicle, there is no unique value for the vehicle mass m_0 and for the magnitude of the impulse represented by the initial velocity V_0. In the case of a two-degree of freedom equipment, the transient analysis can duplicate the maximum equipment response obtained by the DDAM-like input values provided the ratio r of the shock design values is greater than unity. This method, which uses a simple mass to represent the vehicle excited by an impulse, fails to provide a solution when $r \leq 1$.

It may be useful to comment on two special situations that are sometimes mentioned as problem areas for a DDAM analysis. Consider an equipment composed of a very small weight attached to a large weight, as shown in Fig. 11. If $\lambda_1 = \lambda_2$, it can be shown that as the upper mass goes to zero, the modal effective weights approach one-half of the lower weight. Consequently, this equipment model provides stresses and deflections that are considerably higher than a DDAM analysis performed on the lower mass only, i.e., treating the equipment as a single-degree of freedom system composed of m_1 and k_1. The original design shown in Fig. 11 is, of course, a poor one and DDAM is saying just that. One sensible way of avoiding this problem would be to change the properties of the upper mass system before performing the DDAM analysis. For example, earlier work [8,11] examined the model in Fig. 12(a) which led

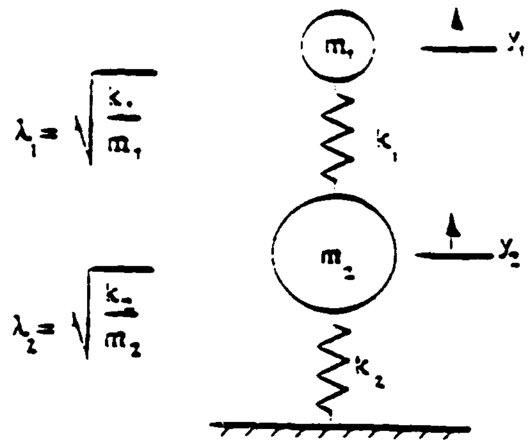

Fig. 11 - Model of an equipment for $m_1 << m_2$.

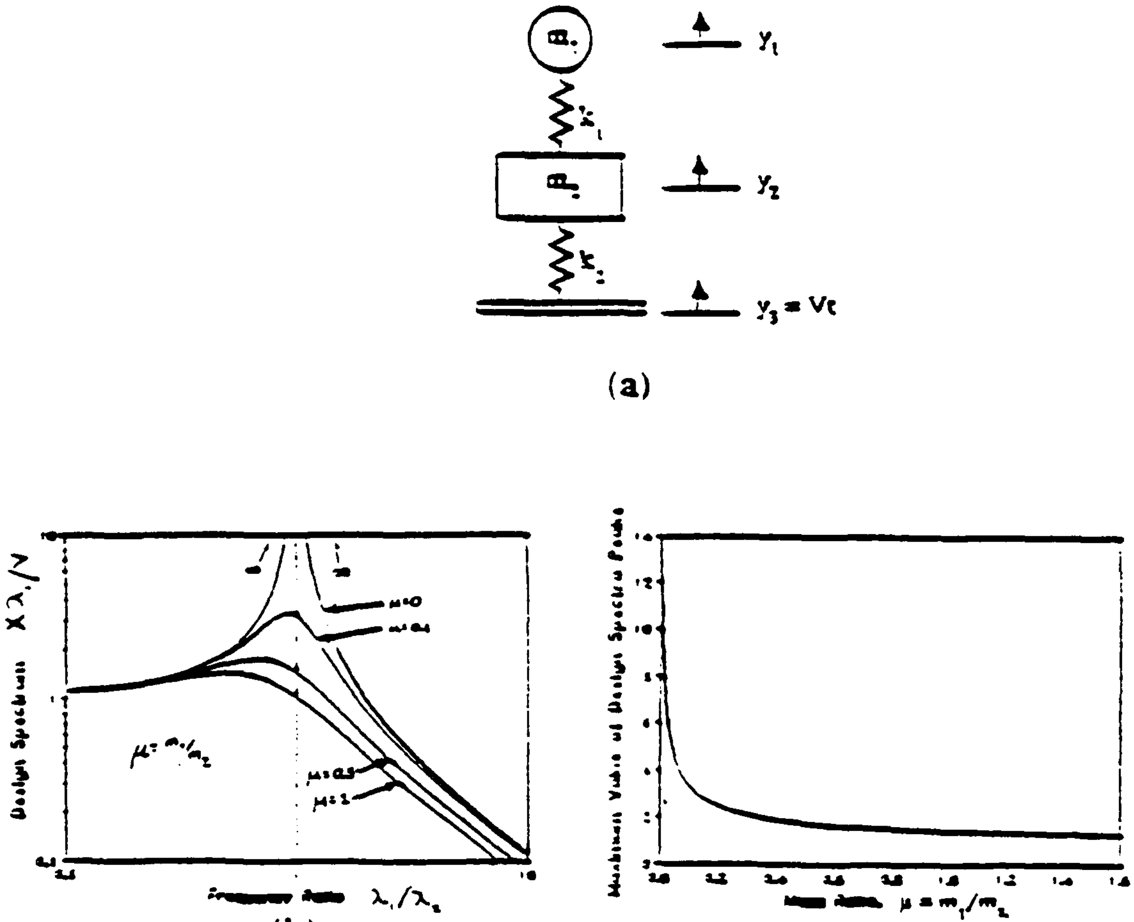

Fig. 12 - Design shock spectra for a two-degree of freedom model

to the response curve measured in terms of the relative displacement between the two masses as shown in Fig. 12(b) for different mass ratios μ. Also, Fig. 12(c) shows a sharp drop of the maximum response as the mass ratio is increased. Thus, a small change in the upper mass m_1 and/or an increase in the stiffness k_1 markedly reduces the response of the upper mass relative to the lower mass. The ensuing analysis should not then dramatically proclaim a bad design for the modified equipment structure.

A final observation on transient analysis is directed to those cases where the equipment fixed base frequencies contain repeated fixed base natural frequencies. It has been demonstrated [12] that the modal effective weights and participation factors that correspond to these repeated roots are zero. Representing the equipment by its modal oscillators which are attached to the vehicle mass presents no problem since the modal oscillators for the repeated frequencies simply will not be present in the transient analysis.

ACKNOWLEDGEMENT

This study is part of a project that is supported by the Office of Naval Research. The content of this paper does not necessarily reflect the position or the policy of the government, and no official endorsement should be inferred.

REFERENCES

1. Belsheim, R.O, and G.J. O'Hara, "Shock Design of Shipboard Equipment, Part I, Dynamic-Design Analysis Method", NRL Report 5545, Sept. 1960. A version also published as NAVSHIPS 250-423-30, May 1961.
2. O'Hara, G.J., "Shock Hardness Assessment of Submarine Equipment; Part V; Survival of Equipment Designed by DDAM", NRL Memorandum Report No. 3942, (U) Confidential Report, Unclassified Title, March, 1979.
3. O'Hara, G.J. and P.F. Cunniff, "Elements of Normal Mode Theory", NRL Report 5917, June 21, 1963.
4. Cunniff, P.F. and G.J. O'Hara, "Normal Mode Theory for Three-Dimensional Motion", NRL Report 6170, Jan. 5, 1965.
5. O'Hara, G.J. and R.O. Belsheim, "Interim Design Values for Shock Design of Shipboard Equipment", NRL Memorandum Report No. 1396, Feb., 1963.
6. A series of private communications among NAVSEA, TRC/UERD, and the University of Maryland on a proposed transient design analysis method, Spring 1990.
7. Remmers, G., "Maurice Biot 50th Anniversary Lecture: The Evolution of Spectral Techniques in Navy Shock Design", Shock and Vibration Bulletin, Part I, 59-70, 1983.
8. O'Hara, G.J. and P.F. Cunniff, "The Shock Spectrum Dip Effect", Jr. of Sound Vibration, Vol. 103, No. 3, 311-321, Dec., 1985.
9. Cunniff, P.F. and G.J. O'Hara, "A Procedure of Generating Shock Design Values", Jr. of Sound and Vibration, Vol. 134, No. 1, 155-164, Oct. 1989.
10. O'Hara, G.J. and L.P. Petak, "The Effect of a Second Mode and Nearby Structures on Shock Design Values", NRL Report 6676, April 5, 1968.
11. O'Hara, G.J., "Shock Spectra and Design Shock Spectra", NRL Report 5386, Sept., 1959.
12. Cunniff, P.F. and R.P. Collins, "Structural Interaction Effects on Shock Spectra", Jr. of the Acoustical Soc. of America, Vol. 43, No. 2, 239-244, Feb., 1968.